

Superselecting target and projectile in high energy scattering

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Overview

Field theory of BFKL pomerons

Symmetry and self-duality

Symmetry breaking

Down to 0-dimensions

Superselection of the target

Based on work done with Sergey Bondarenko

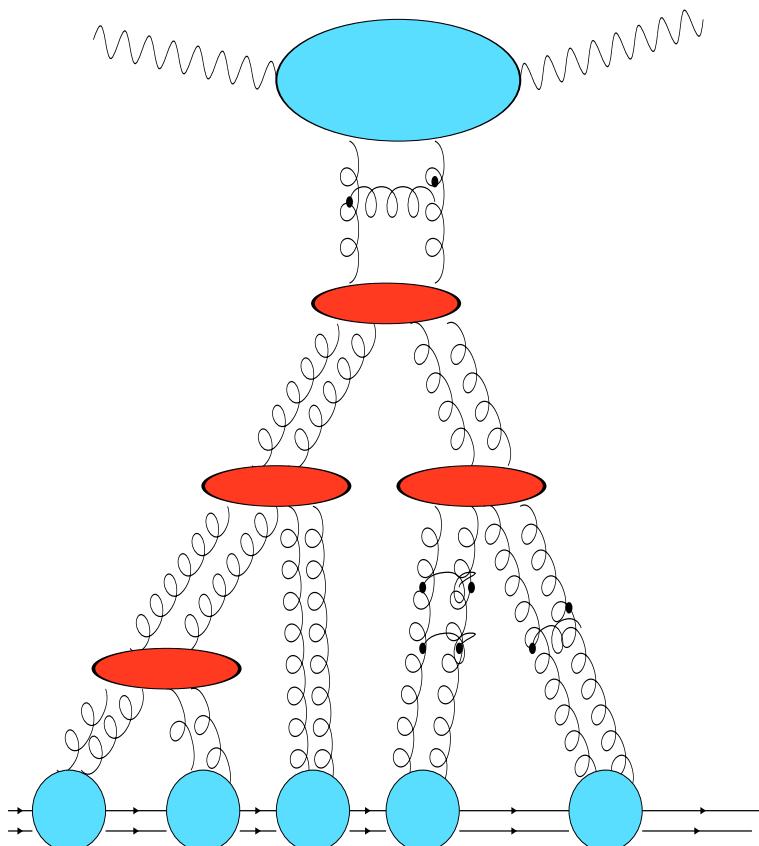
Saturation in asymmetric scattering: BK/JIMWaLK equation

[Balitsky,Kovchegov],[Jalilian-Marian,Iancu,McLerran, Leonidov,Weigert]

Propagation of a dilute projectile through a dense target at large energy – at the LL1/ x : BFKL evolution of amplitude tempered by unitarity corrections

Enhancement + Correlations + Rescattering

Target frame: in the large N_c limit BFKL pomeron fan diagrams – gluon recombination at large density



$$\frac{\partial N(\mathbf{x}, \mathbf{y}; Y)}{\partial Y} = \frac{\bar{\alpha}_s}{2\pi} \int d^2 z \frac{(\mathbf{x} - \mathbf{y})^2}{(\mathbf{x} - \mathbf{z})^2 (\mathbf{z} - \mathbf{y})^2} \times$$

$$[N(\mathbf{x}, \mathbf{z}; Y) + N(\mathbf{z}, \mathbf{y}; Y) - N(\mathbf{x}, \mathbf{y}; Y) - N(\mathbf{x}, \mathbf{z}; Y)N(\mathbf{z}, \mathbf{y}; Y)]$$

BK equation in momentum space

$$\frac{\partial N(k, Y)}{\partial Y} = \chi_{\text{BFKL}} \left(1 + k^2 \partial_{k^2} \right) N(k, Y) - N^2(k, Y)$$

For large k / small r – BFKL-like growth with y .
For small k / large r – saturation.

Unitarity is not violated

Kwieciński–Kutak formulation of BK

Dipole density in momentum space:

$$\tilde{N}(y, k^2) = \int \frac{d^2\mathbf{r}}{2\pi} \exp(-i\mathbf{k} \cdot \mathbf{r}) \frac{N(y, r)}{r^2}$$

Forward dipole scattering amplitude $N(y, r) \propto \int \frac{d^2k}{k^4} f(y, k^2) [1 - \exp(-i\mathbf{k}\mathbf{r})]$

Unintegrated gluon density (t -channel) $f(y, k^2) \propto k^4 \nabla_k^2 \tilde{N}(y, k^2)$

BK equation $\partial_y f(y, k^2) = \bar{\alpha}_s k^2 \int \frac{da^2}{a^2} \left[\frac{f(y, a^2) - f(y, k^2)}{|a^2 - k^2|} + \frac{f(y, k^2)}{[4a^4 + k^4]^{\frac{1}{2}}} \right]$

$$-2\pi \bar{\alpha}_s^2 \left[k^2 \int_{k^2} \frac{da^2}{a^4} f(y, a^2) \int_{k^2} \frac{db^2}{b^4} f(y, b^2) + f(y, k^2) \int_{k^2} \frac{da^2}{a^4} \log \left(\frac{a^2}{k^2} \right) f(y, a^2) \right]$$

Limitations of BK and need to improve

BK is well founded for very **asymmetric** scattering like $\gamma^* - \text{Nucleus}$, DIS

Quantum effects (**pomeron loops**) are absent

Need to describe multiple scattering and production in **symmetric** situation like **heavy ion collisions** at RHIC and **pp scattering** at the LHC

Three complementary ways to go beyond BK:

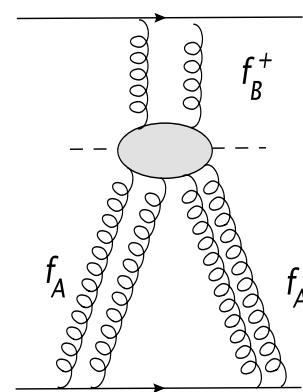
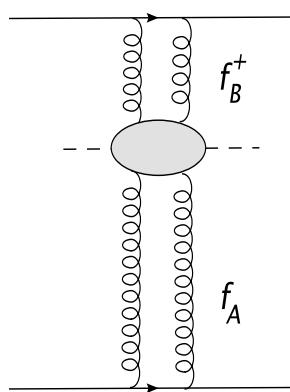
Effective Field Theory approach \longrightarrow Path integral: $\int [D\phi_\alpha] \exp(-S[\phi_\alpha, \partial\phi_\alpha])$

Stochastic formulation \longrightarrow Langevin equation: $\partial_Y N = A(N) + B(N)\nu(Y)$

Hamiltonian formulation \longrightarrow Generating functional: $\partial_Y \Psi[Y; u] = (\mathcal{H} \otimes \Psi)[Y; u]$

Field theoretical formulation of BK

[Bondarenko,LM]



Amplitudes of the diagrams:

$$\int \frac{d^2 k}{k^4} f_A(y, k^2) f_B^\dagger(y, k^2)$$

$$\int \frac{d^2 k}{k^4} \int \frac{d^2 a}{a^4} \int \frac{d^2 b}{b^4} V_{3P}(\mathbf{k}; \mathbf{a}, \mathbf{b}) f_B^\dagger(y, k^2) f_A(y, a^2) f_A(y, b^2)$$

BFKL Green's function

$$\int \frac{d^2 k}{k^4} \frac{d^2 k'}{k'^4} f_A(k'^2) \mathcal{G}(Y; k'^2, k^2) f_B^\dagger(k^2) = \int \frac{d^2 k}{k^4} \frac{d^2 k'}{k'^4} f_A(k'^2) \frac{1}{\partial_y - \hat{K}_0(k'^2, k^2)} f_B^\dagger(k^2)$$

Effective action $\mathcal{A}[f, f^\dagger; Y] \propto \int_0^Y dy \left\{ \mathcal{L}_0[f, f^\dagger] + \mathcal{L}_3[f, f^\dagger] + \mathcal{L}_E[f, f^\dagger] \right\}$

$$\mathcal{L}_0[f, f^\dagger] = \frac{1}{2} \left[f(y) \otimes \overleftrightarrow{\partial}_y \otimes f^\dagger(y) \right] + f^\dagger(y) \otimes \mathcal{K}_0 \otimes f(y)$$

$$\mathcal{L}_3[f, f^\dagger] = -2\pi\alpha_s^2 \langle f^\dagger(y) | V_{3P} | f(y) \otimes f(y) \rangle$$

$$\mathcal{L}_E[f, f^\dagger] = f_A^\dagger(y) \otimes f(y) + f^\dagger(y) \otimes f_B(y)$$

Target – projectile symmetry

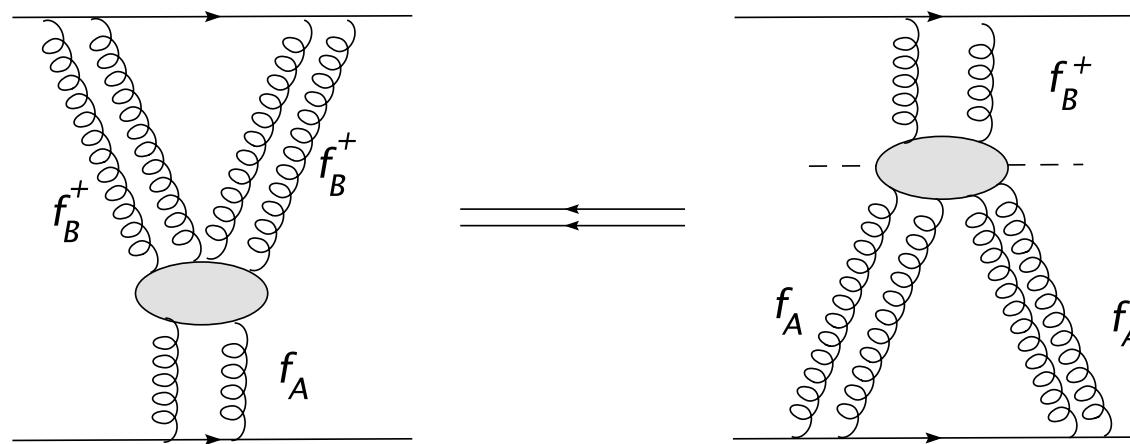
[Kovner,Lublinsky]

Direction of rapidity evolution is arbitrary

Free to choose target and projectile

Action symmetric w.r.t. target \longleftrightarrow projectile

$$y \rightarrow -y, \quad f(y, k^2) \longleftrightarrow f^\dagger(Y - y, k^2)$$



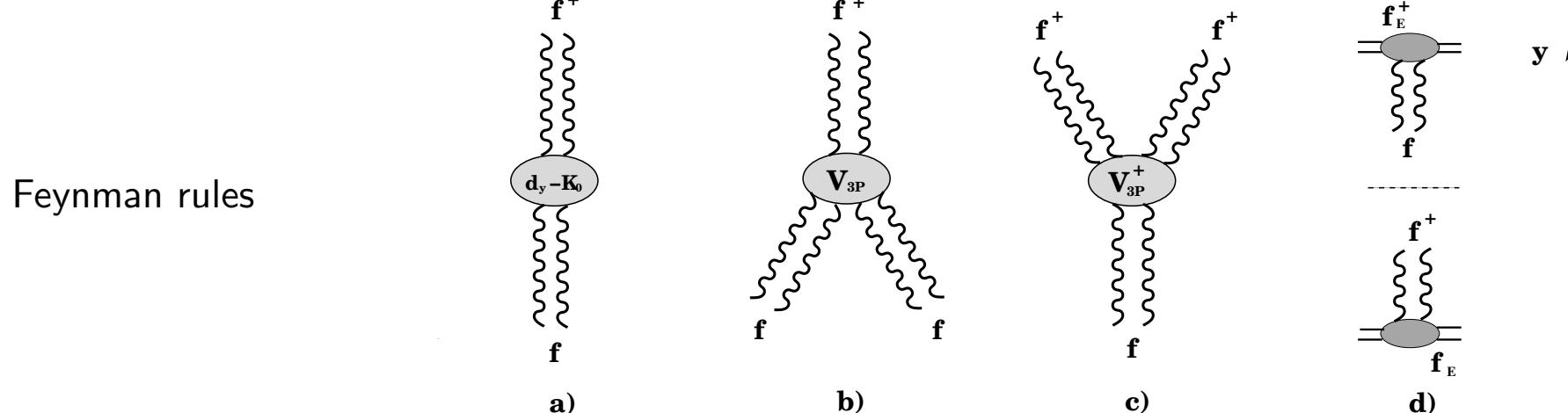
$$\langle f^\dagger(y) \otimes f^\dagger(y) | V_{3P}^\dagger | f(y) \rangle = \langle f^\dagger(y) | V_{3P} | f(y) \otimes f(y) \rangle^\dagger$$

Canonical structure — Gribov fields

$$\frac{\delta \mathcal{L}_0[f, f^\dagger]}{\delta(\partial_y f^\dagger(y, k^2))} = \frac{1}{k^4} f(y, k^2), \quad \frac{\delta \mathcal{L}_0[f, f^\dagger]}{\delta(\partial_y f(y, k^2))} = -\frac{1}{k^4} f^\dagger(y, k^2)$$

Canonical commutator $[-if(y, k^2), -if^\dagger(y, k'^2)] = k^4 \delta(k^2 - k'^2)$

Gribov fields $\frac{-if(y, k^2)}{k^2} \longrightarrow \psi, \quad \frac{-if^\dagger(y, k^2)}{k^2} \longrightarrow \psi^\dagger$



f and f^\dagger are canonically conjugated \Rightarrow Self-duality

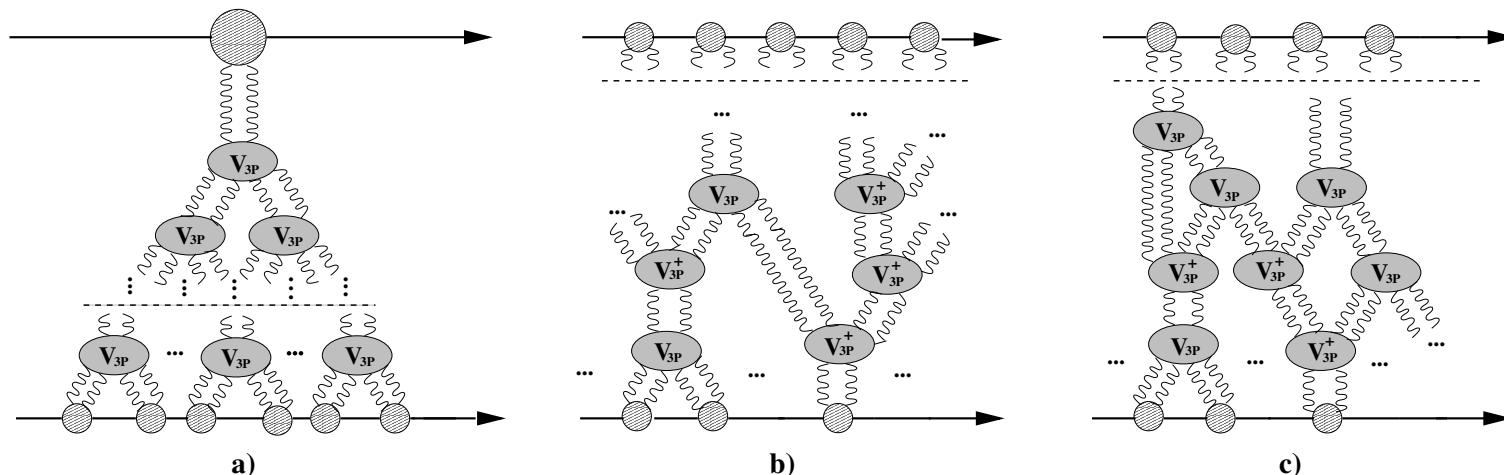
Semi-classical limit (valid for large nuclei)

Effective action $\mathcal{A}[f, f^\dagger; Y] \propto \int_0^Y dy \left\{ \mathcal{L}_0[f, f^\dagger] + \mathcal{L}_3[f, f^\dagger] + \mathcal{L}_3^\dagger[f, f^\dagger] + \mathcal{L}_E[f, f^\dagger] \right\}$

$S - \text{matrix: } S(Y; f_A, f_B^\dagger) = \int_{f_A, f_B^\dagger} [Df Df^\dagger] \exp(-\mathcal{A}[f, f^\dagger; Y])$

Classical trajectories $\bar{f}(y, k^2)$ and $\bar{f}^\dagger(y, k^2)$: $\delta \mathcal{A}[\bar{f}, \bar{f}^\dagger; Y] = 0$

$$S(Y; f_A, f_B^\dagger) = \sum_\alpha \Delta_\alpha \exp(-\mathcal{A}[\bar{f}_\alpha, \bar{f}_\alpha^\dagger; Y])$$



Equations of motion – Braun equations in KK representation

Mutual absorption of the fields

$$\begin{aligned} \partial_y f(y, k^2) = & \text{BK}(f, k^2) - 2\pi\alpha_s^2 \left[2 \int_0^{k^2} \frac{da^2}{a^4} a^2 f(y, a^2) \int_{a^2} \frac{db^2}{b^4} f^\dagger(y, b^2) + \right. \\ & \left. + f(y, k^2) \int_{k^2} \frac{da^2}{a^4} \log\left(\frac{a^2}{k^2}\right) f^\dagger(y, a^2) + \int_0^{k^2} \frac{da^2}{a^4} f(y, a^2) f^\dagger(y, a^2) \log\left(\frac{k^2}{a^2}\right) \right] \end{aligned}$$

and

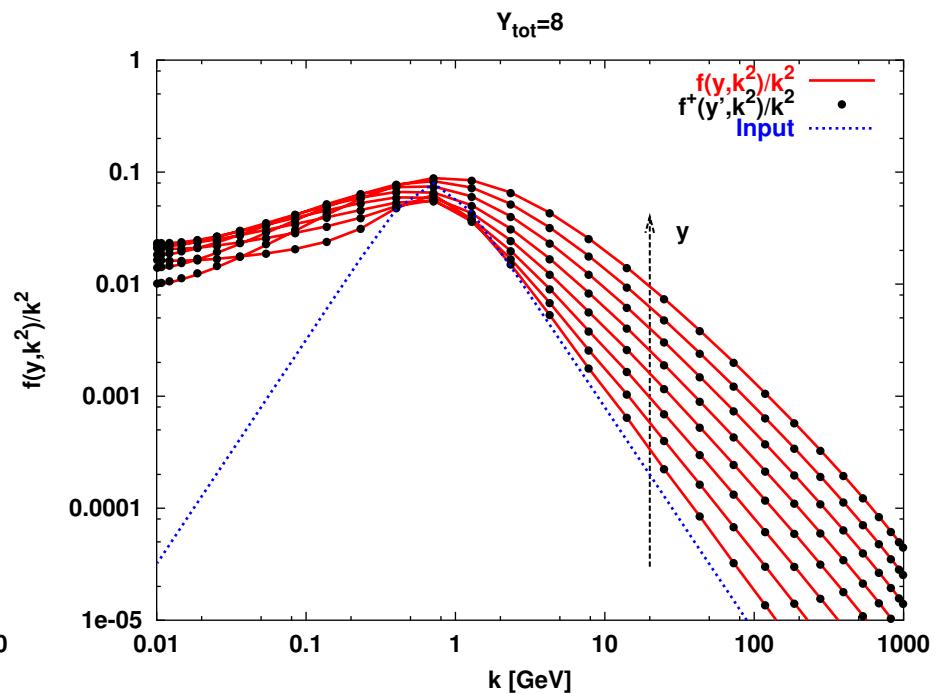
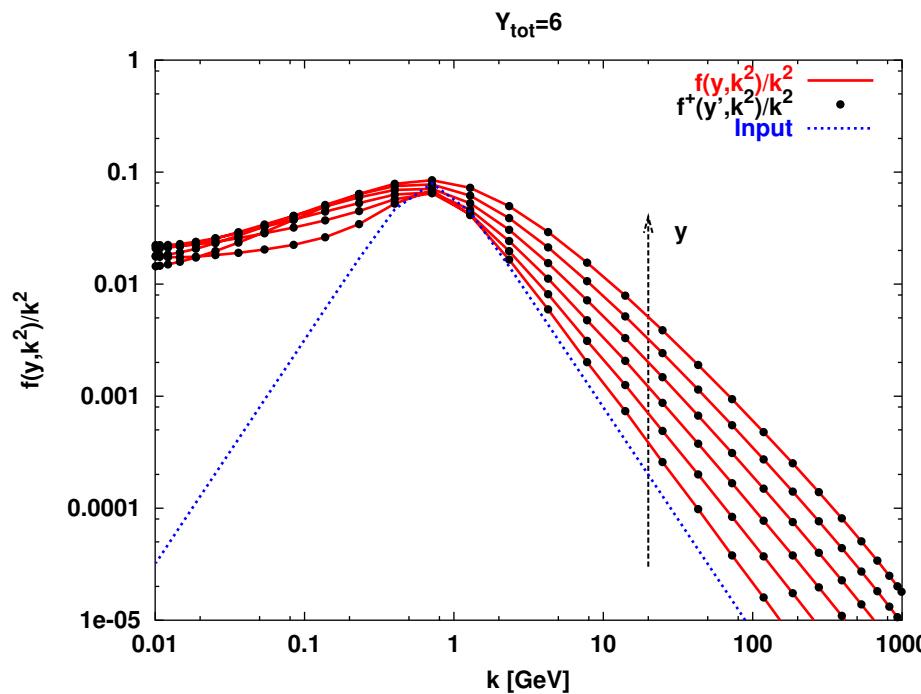
$$\begin{aligned} -\partial_y f^\dagger(y, k^2) = & \text{BK}(f^\dagger, k^2) - 2\pi\alpha_s^2 \left[2 \int_0^{k^2} \frac{da^2}{a^4} a^2 f^\dagger(y, a^2) \int_{a^2} \frac{db^2}{b^4} f(y, b^2) + \right. \\ & \left. + f^\dagger(y, k^2) \int_{k^2} \frac{da^2}{a^4} \log\left(\frac{a^2}{k^2}\right) f(y, a^2) + \int_0^{k^2} \frac{da^2}{a^4} f^\dagger(y, a^2) f(y, a^2) \log\left(\frac{k^2}{a^2}\right) \right] \end{aligned}$$

Symmetric two-point boundary conditions corresponding to eikonal sources:

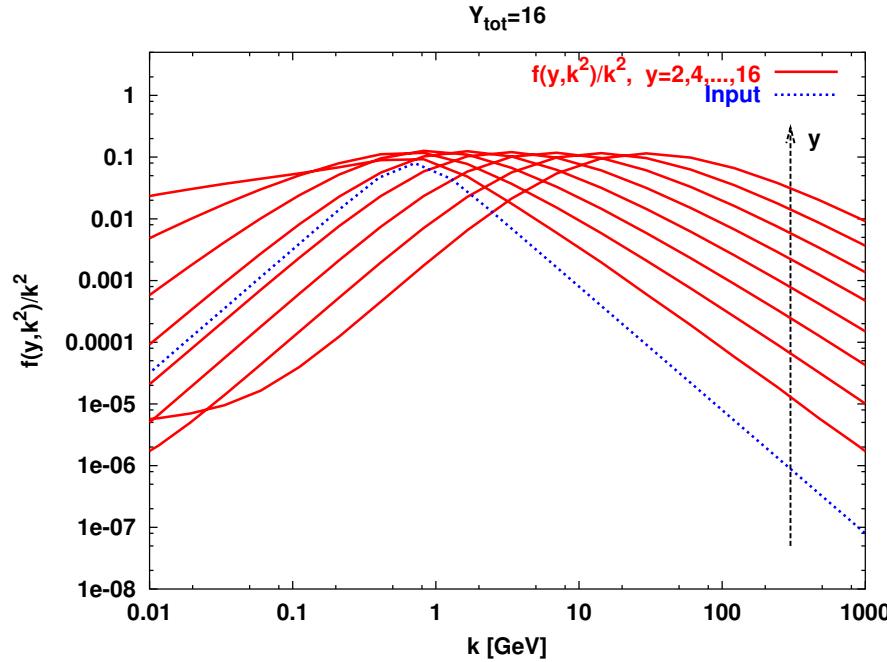
$$f(y = 0, k^2) = f_A(k^2), \quad f^\dagger(y = Y, k^2) = f_B^\dagger(k^2) = f_A(k^2)$$

Symmetric solutions of Braun equations

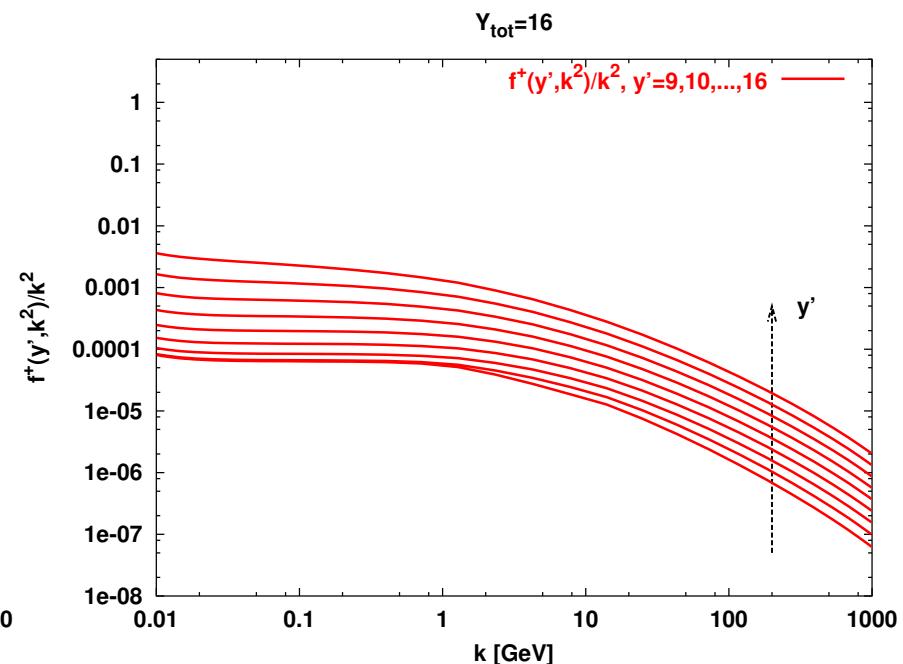
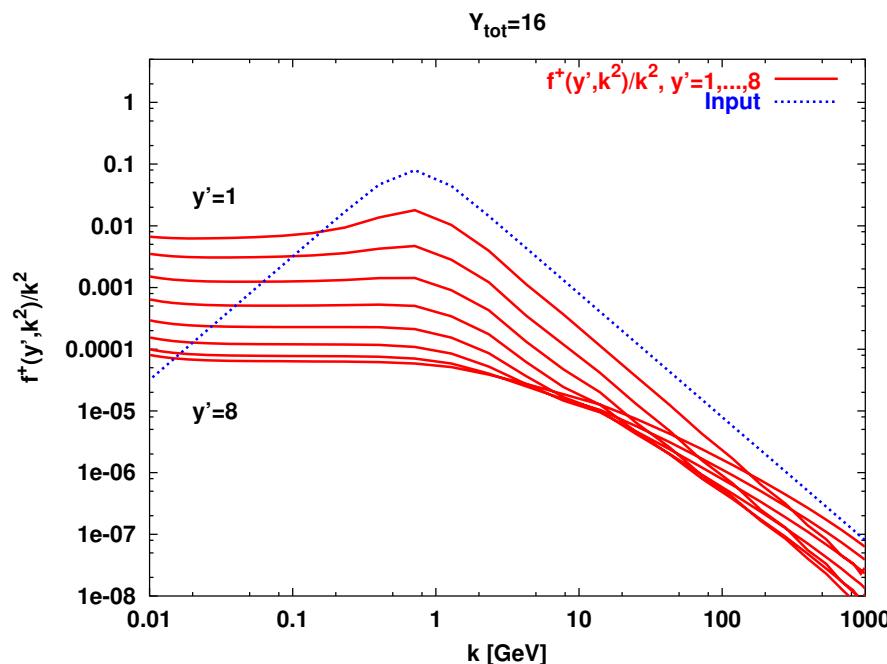
$$f(y, k^2)/k^2 = f^\dagger(Y - y, k^2)/k^2 \quad \text{for } Y = 6 \text{ and } Y = 8$$



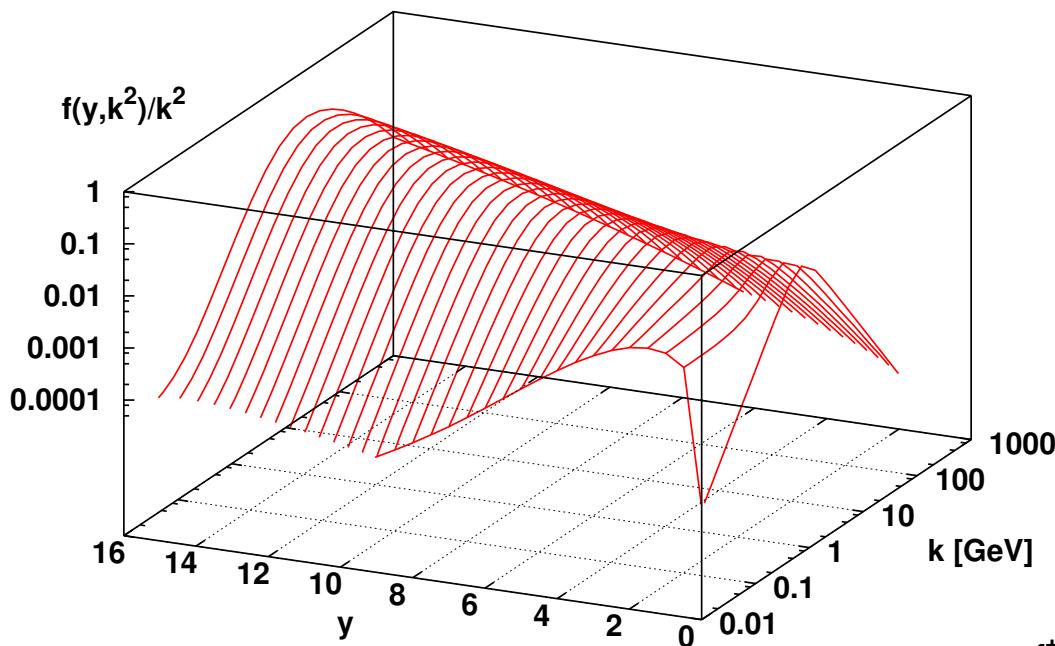
Spontaneous Symmetry Breaking at classical level



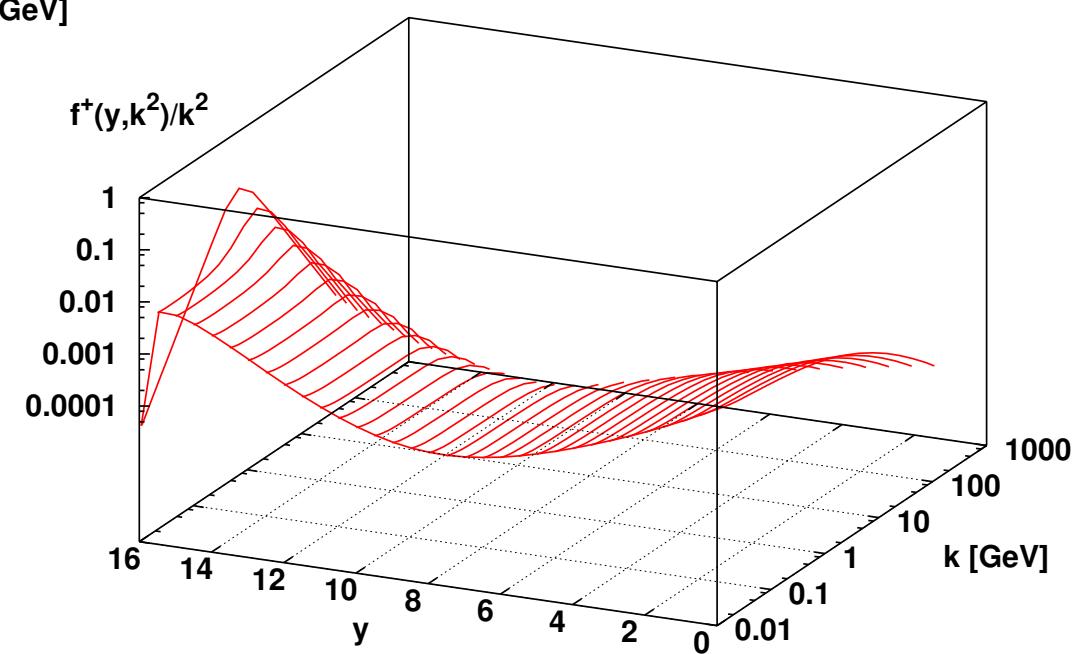
Above critical rapidity Y_c – at $Y = 16$
 $f(y, k^2)$ (left) larger than
 $f^\dagger(Y - y, k^2)$ (down)
by three orders of magnitude



$\gamma_{\text{tot}} = 16$



$\gamma_{\text{tot}} = 16$

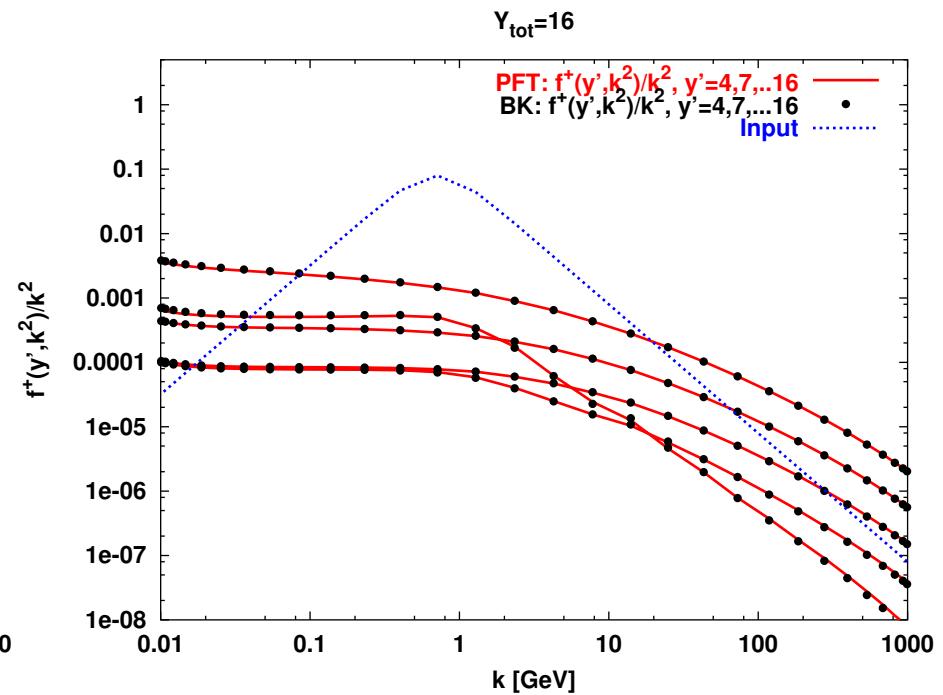
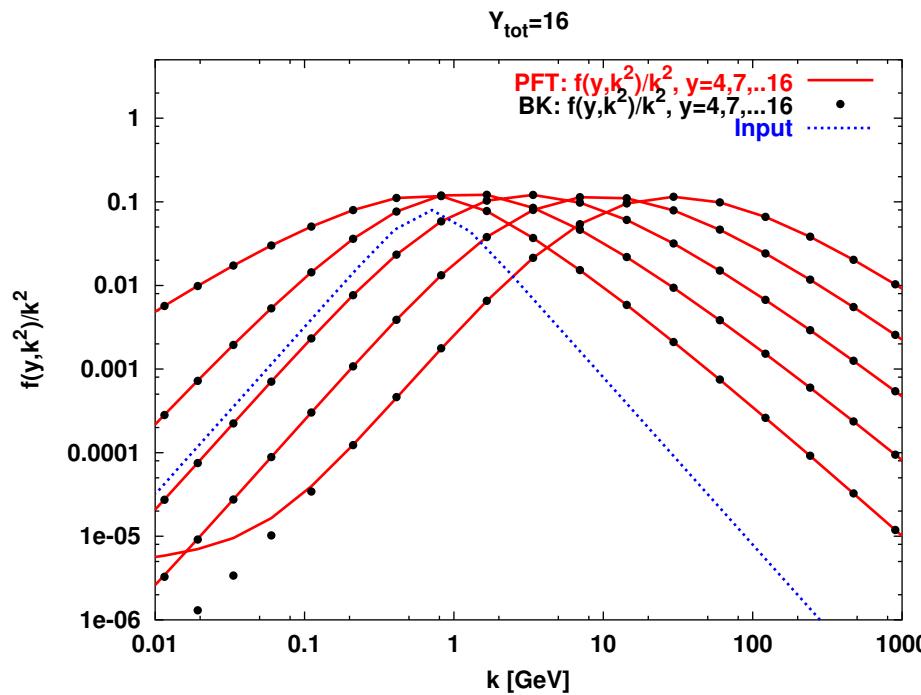


Fan dominance

Solutions of Braun equations vs BK solutions

$$f(y, k^2)$$

$$f^\dagger(y, k^2)$$



Detour to 0-dimensions

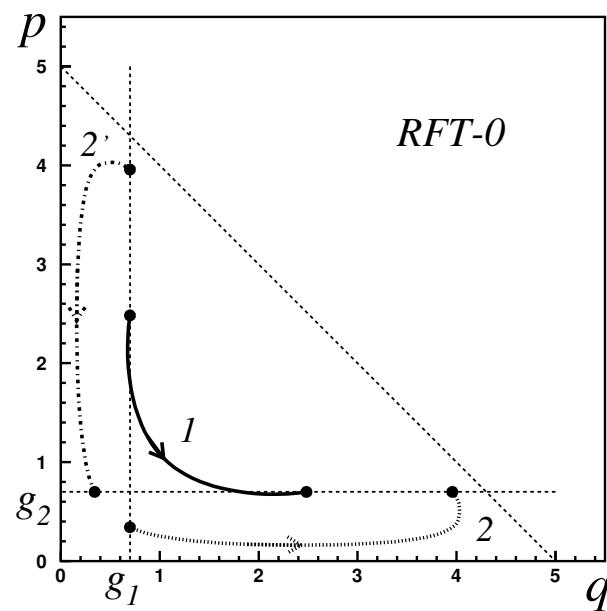
Formulation

$$\mathcal{L}_{\text{RFT-0}} = \frac{1}{2} q \partial_y p - \frac{1}{2} p \partial_y q + \mu q p - \lambda q (q + p) p + p(y) q_0(y) + p_0(y) q(y)$$

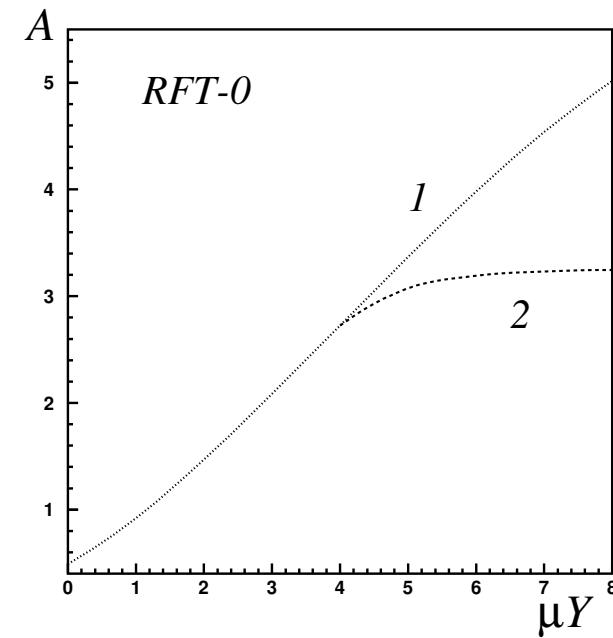
Equations of motion

$$\partial_y q = \mu q - \lambda q^2 - 2\lambda q p; \quad -\partial_y p = \mu p - \lambda p^2 - 2\lambda q p$$

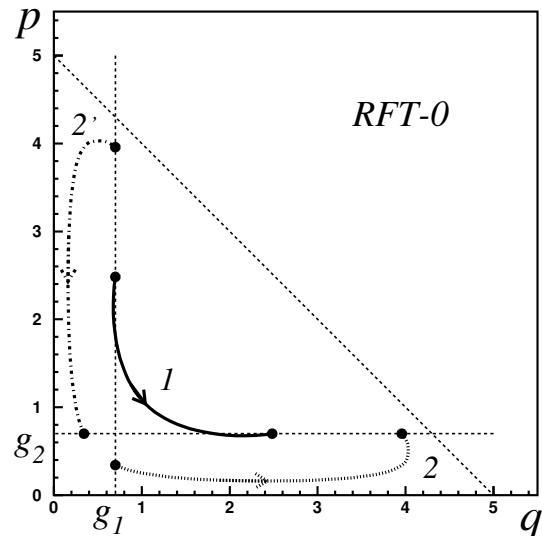
Phase space and trajectories



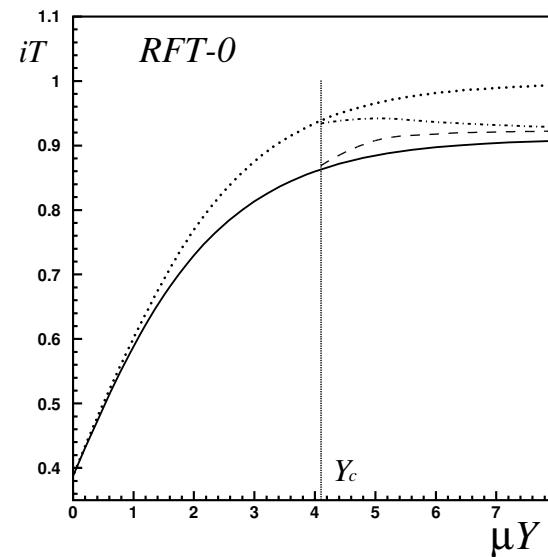
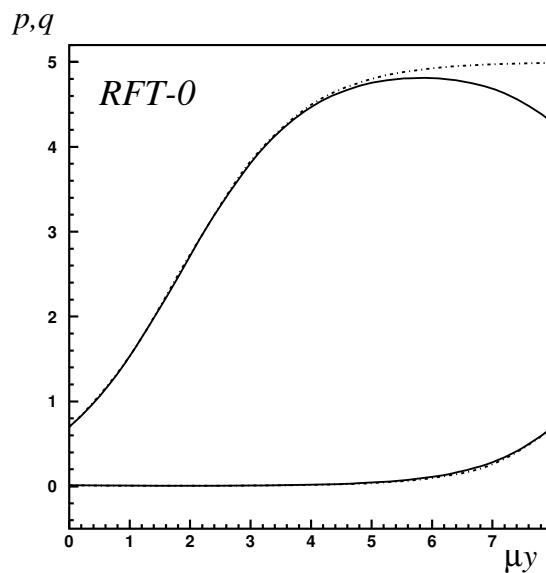
Action



Semi-classical vs exact, fan dominance



Fan dominance:



Symmetric set of solutions

$$\bar{q}'_2(y; g, g) = \bar{p}_2(Y - y; g, g)$$

$$\bar{p}'_2(y; g, g) = \bar{q}_2(Y - y; g, g).$$

$$\begin{aligned} S(Y; g_1, g_2) \simeq & \\ -\exp \{ -\mathcal{A}_{RFT-0}[\bar{q}_1, \bar{p}_1; Y] \} & \\ +\exp \{ -\mathcal{A}_{RFT-0}[\bar{q}_2, \bar{p}_2; Y] \} & \\ +\exp \{ -\mathcal{A}_{RFT-0}[\bar{q}'_2, \bar{p}'_2; Y] \} & \end{aligned}$$

Asymptotic answer for sources
 g_1 and g_2 :

$$T \sim [1 - \exp(-\rho g_1)][1 - \exp(-\rho g_2)]$$

Superselecting target and projectile

Symmetric set of solutions

$$f'_2(y, k^2) = f_2^\dagger(Y - y; k^2), \quad f'^\dagger_2(y, k^2) = f_2^\dagger(Y - y, k^2)$$

Target–projectile symmetry of the S -matrix

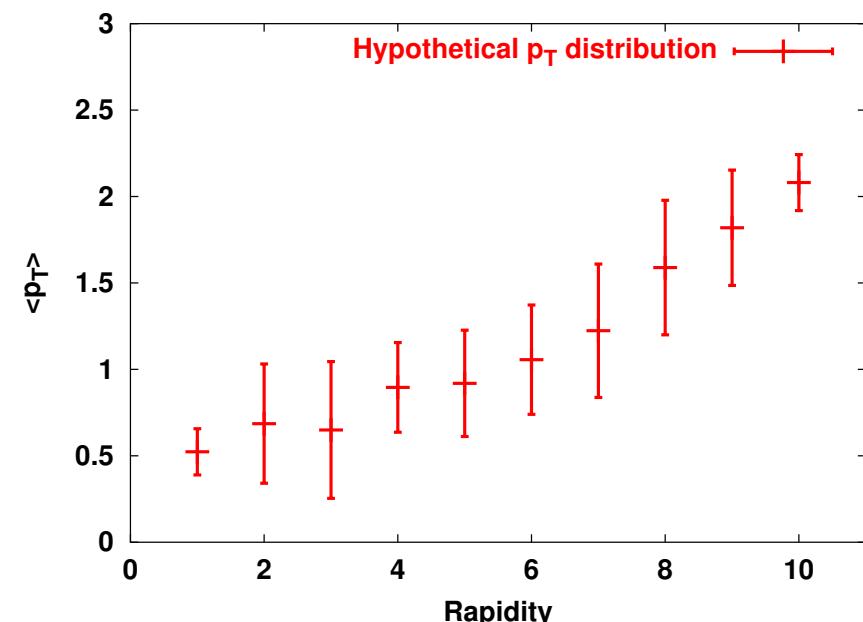
$$S(Y) \simeq -\exp \left\{ -\mathcal{A}[f_1, f_1^\dagger; Y] \right\} + \exp \left\{ -\mathcal{A}[f_2, f_2^\dagger; Y] \right\} + \exp \left\{ -\mathcal{A}[f'_2, f'^\dagger_2; Y] \right\}$$

Classical measurement of the event: for asymmetric trajectories saturation scale $Q_s(y)$ monotonic between target and projectile

Super-selection of asymmetric classical trajectory?

Proposal: Analysis of $\langle p_T(y) \rangle$ on the event-by-event basis

Possible: uncorrelated domains in impact parameter plane



Conclusions

- Field theory of interacting QCD pomerons in momentum space was constructed incorporating both pomeron mergings and splittings
- Degrees of freedom are pomeron fields related to unintegrated gluon densities and the action is self-dual
- The theory was solved in the semi-classical approximation for symmetric two-side boundary conditions (Braun equations)
- Above critical rapidity Y_c symmetry between target and projectile is spontaneously broken at the classical level
- Superselection of asymmetric configurations by classical measurements is suggested
- Possible asymmetric particle distributions in heavy ion collisions (event-by-event)